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The radiative return at Φ- and B-factories: A status report⋆

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Abstract. The measurement of the pion form factor and, more generally, of the cross section for electron–positron annihilation into hadrons through the radiative return has become an important task for high luminosity colliders such as the Φ- or B-meson factories. For detailed understanding and analysis of this reaction, the construction of a Monte Carlo program, PHOKHARA, has been undertaken. Version 2.0 was based on a next-to-leading order (NLO) treatment of the corrections from initial-state radiation (ISR) and leading order final-state radiation (FSR). The most recent extension of PHOKHARA (version 3.0), which incorporates NLO corrections to FSR and the impact of combined ISR and FSR on selected distributions, is presented.

Electron–positron annihilation into hadrons is one of the basic reactions of particle physics, crucial for the understanding of hadronic interactions. At high energies, around the Z resonance, the measurement of the inclusive cross section and its interpretation within perturbative QCD [1] give rise to one of the most precise and theoretically founded determinations of the strong coupling constant α_s [2]. Also, measurements in the intermediate energy region, between 3 GeV and 11 GeV can be used to determine α_s and at the same time give rise to precise measurements of charm and bottom quark masses [3]. The low energy region is crucial for predictions of the hadronic contributions to a_μ, the anomalous magnetic moment of the muon, and to the running of the electromagnetic coupling from its value at low energy up to M_Z (for reviews see e.g. [4,5,6,7,8]). Last, but not least, the investigation of the exclusive final states at large momenta allows for tests of our theoretical understanding of form factors within the framework of perturbative QCD.

The recent advent of Φ- and B-meson factories with their enormous luminosities allows us to exploit the radiative return to explore the whole energy region from threshold up to the nominal energy of the collider. Photon radiation from the initial state reduces the cross section by a factor $\mathcal{O}(\alpha/\pi)$. However, this is easily compensated by the enormous luminosity of 'factories' and the advantage of performing the measurement over a wide range of energies in one homogeneous data sample [9] (for an early proposal along these lines, see [10]). In principle the reaction $e^+e^- \to \gamma +$ hadrons receives contributions from both initial- and final-state radiation. Only the former is of interest for the radiative return; the latter has to be eliminated by suitably chosen cuts. The proper analysis thus requires the construction of Monte Carlo event generators. The event generator EVA was based on a leading order treatment of ISR and FSR, supplemented by an approximate inclusion of additional collinear radiation based on structure functions and included two-pion [9] and four-pion final states [11]. Subsequently the event generator PHOKHARA was developed; it is based on a complete next-to-leading order (NLO) treatment of ISR [12,13,14,15]. In its version 2.0 it included ISR at NLO and FSR at leading order (LO) for $\pi^+\pi^-$ and $\mu^+\mu^-$ final states and four-pion final states without FSR.

Recent preliminary experimental results indeed demonstrate the power of the method and seem to indicate that a precision of one per cent or better is within reach [16]. In view of this progress a further improvement of theoretical understanding and MC-programs is necessary. The most recent version of PHOKHARA, version 3.0 [17], allows for the 'simultaneous' emission of one photon from the initial and one photon from the final state, requiring only one of them to be hard. This includes in particular the radiative return to $\pi^+\pi^-(\gamma)$ and thus allows the measurement of the (one-photon) inclusive $\pi^+\pi^-$ cross section.

The issue of photon radiation from the final states is closely connected to the question of $\pi^+\pi^-(\gamma)$ contributions to a_μ (for related discussions, see e.g. [6,18]). Soft photon emission is clearly described by the point-like pion model. Hard photon emission, with $E_\gamma \geq \mathcal{O}(100 \text{ MeV})$, however, might be sensitive to unknown hadronic physics. Therefore the size of virtual, soft and hard corrections has to be studied separately. It will be argued that contributions from the hard region, above 100 MeV, are small with respect to the present experimental and theory-induced uncertainty.

Let us start with the process in leading order:

$$e^+ \, e^- \to \pi^+(p^+) \, \pi^-(p^-) \, \gamma \, . \tag{1}$$

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The amplitude of interest describes radiation from the initial state (ISR). It is proportional to the pion form factor, evaluated at $Q^2 = (p^+ + p^-)^2$. However, radiation from the charged pions (FSR) evidently leads to the same final state and must be suppressed and controlled with sufficient precision. A variety of methods are described in more detail in [9,13,15,17].

The fully differential cross section describing photon emission can be split into three pieces

$$d\sigma = d\sigma_{\rm ISR} + d\sigma_{\rm FSR} + d\sigma_{\rm INT} \ , \qquad (2)$$

which originate from the squared ISR and FSR amplitudes respectively, plus the interference term. They depend on two Dalitz variables, which characterize the energies of π^+ and π^- and the photon, and on the three Euler angles describing the orientation of the $\pi^+\pi^-\gamma$ production plane in the centre-of-mass system (cms).

The interference term, $d\sigma_{\rm INT}$, is antisymmetric under the exchange of π^+ and π^- or e^+ and e^-. This allows us to test the model for FSR.

The charge-symmetric pieces can be separated on the basis of their markedly different angular distribution, either by fitting the two components [17] or by employing suitable cuts.

Let us summarise the main points of the 'leading order' discussion [17]:

1. The cross sections for photon emission from ISR and FSR can be disentangled as a consequence of the marked difference in angular distributions between the two processes. This observation is completely general and does not rely on any model like sQED for FSR. This allows us to measure the cross section for $\sigma(e^+e^- \to \gamma^* \to \pi^+\pi^-\gamma)$ directly for fixed s as a function of E_γ, an important ingredient in the analysis of hadronic contributions to a_μ. Alternatively we can employ suitable angular cuts to separate the two components.
2. Various charge-asymmetric distributions can be used for independent tests of the FSR model amplitude, with the forward–backward asymmetry as simplest example. Typically the charge asymmetry is large in the region where ISR and FSR are comparable in size and small when the separation is clean and simple from general considerations. A typical case for this second possibility is the radiative return at $\sqrt{s} = 10.52$ GeV, at the B-meson factories, where the $\pi^+\pi^-\gamma$ final state is completely dominated by ISR.

At present hadronic contributions to a_μ are known to roughly half to one percent. Photonic corrections to final states with hadrons thus start to become relevant. Qualitatively the order of magnitude of this effect can be estimated either by using the quark model with $m_u \approx m_d \approx m_s \approx 180$ MeV (adopted to describe the lowest order contribution [19]), or with $m_u \approx m_d \approx m_s \approx 66$ MeV (adopted to describe the lowest order contribution to $\alpha(M_Z)$) or by using $\pi^+\pi^-$ as dominant intermediate hadronic state plus photons coupled according to sQED. The three estimates

$$\begin{aligned} \delta a_\mu(\text{quark}, \gamma, m_q = 180 \text{ MeV}) &= \ 1.880 \times 10^{-10} \ , \\ \delta a_\mu(\text{quark}, \gamma, m_q = 66 \text{ MeV}) &= \ 8.577 \times 10^{-10} \ , \\ \delta a_\mu(\pi^+\pi^-, \gamma) &= \ 4.309 \times 10^{-10} \ , \end{aligned} \qquad (3)$$

are comparable in magnitude and may be relevant at the present level of precision.

In [17] it has been shown that the measurement of $R(\pi^+\pi^-(\gamma))$ is indeed possible with the technique of the radiative return. The following strategy for the exclusive measurement was proposed:

The contribution from $R^{\rm V+R}(\pi^+\pi^-(\gamma), E^{\rm cut})$ as a function of $E^{\rm cut}$ can be measured by analysing final states where hard-photon radiation is suppressed by suitable chosen cuts on energy and collinearity of the pions. The $E^{\rm cut}$ dependence of $R^{\rm H}(\pi^+\pi^-\gamma, E^{\rm cut})$ can be measured by collecting $\pi^+\pi^-\gamma$ final states (and correcting of course for ISR). Alternatively we may correct for (unmeasured) hard photon events by employing a model like sQED, which has to be checked experimentally — at least for a few selected kinematic configurations.

Using sQED as a model, one finds [17] that contributions from hard photon radiation, with a cut at 100 MeV, are still small relative to the present uncertainty $\delta a_\mu^{\rm had,LO} = 7 \times 10^{-10}$ [7,8]. However, cuts on the photon energy around 50 MeV or below might well lead to important shifts in a_μ. Of course we have assumed that hard radiation is not grossly underestimated by sQED, an assumption to be tested by experiment.

Intuitively it should also be possible to exploit the radiative return for the extraction of FSR for arbitrary invariant mass $\sqrt{s'}$ of the $\pi^+\pi^-\gamma$ system through the reaction

$$e^+e^- \to \gamma\gamma^*(\to \pi^+\pi^-\gamma) \ . \qquad (4)$$

The implementation of this two-step process has been achieved in of the newest version of PHOKHARA (PHOKHARA 3.0).

The impact of the new corrections on a few selected distributions has been discussed in [17]. Before entering the discussion, let us recall the meaning of various abbreviations, which will be used in the following. ISRNLO corresponds to the ISR cross section calculated at NLO without any FSR. IFSLO includes in addition FSR at LO. Finally, IFSNLO stands for ISR and FSR at NLO as implemented in the new version of PHOKHARA (version 3.0).

For a cms energy of 1.02 GeV, relevant for the KLOE experiment, the IFSNLO correction to the cross section is relatively big at low Q^2, if no cuts are applied (Fig. 1a). Below the ρ resonance, they grow from zero at the resonance to 10% of the IFSLO cross section near the production threshold, while they remain small above the ρ resonance. This is due to the fact that the ISR leading to the ρ meson is strongly enhanced. Subsequently the ρ decays into $\pi^+\pi^-\gamma$, with a large contribution from the region where Q^2, the invariant mass of the $\pi^+\pi^-$, is low. Of course this is smeared by the width of the ρ, but the

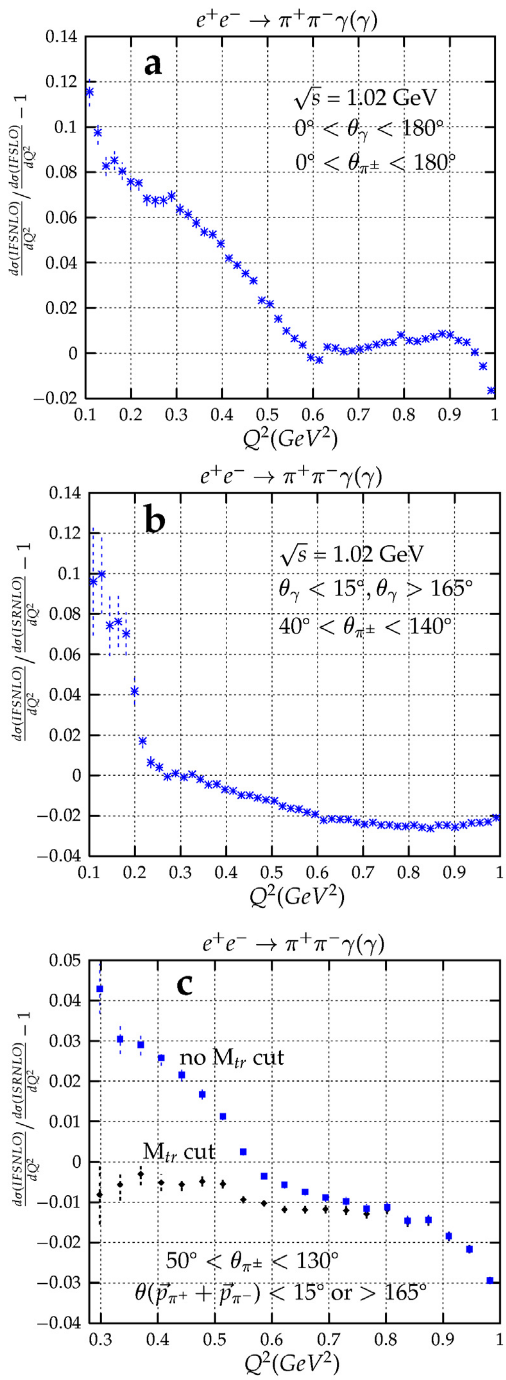

Fig. 1. Comparison of the Q^2 differential cross sections for $\sqrt{s} = 1.02$ GeV: IFSNLO contains the complete NLO contribution, while IFSLO has FSR only at LO. The pion and photon(s) angles are not restricted in **a**, and restricted in **b**. In **c** the cuts are imposed on the missing momentum direction and the track mass (see text for description)

above discussion remains valid and the contribution from the newly implemented diagrams, through the reaction $e^+e^- \to \gamma\rho(\to \gamma\pi^+\pi^-)$, is sizeable.

The effect of several 'standard cuts' at $\sqrt{s} = 1.02$ GeV is shown in Figs. 1b and 1c. The sensitivity of the newly implemented contributions to these cuts can be exploited to test, control or experimentally eliminate FSR at NLO. It is relatively easy to find cuts that lower the correction to a level of 2%–3%. In fact all the cuts which were used to eliminate FSR at LO are also effective here. The standard KLOE cut on the track mass [20] reduces FSR further, to less than 1% for most of the Q^2 range, apart of the high values of Q^2, where the corrections remain at the level of 2%-3% (Fig. 1c, lower curve). The behaviour of the additional contribution at 10.52 GeV is qualitatively similar.

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